

Chapter 3. Trigonometric Functions

Question-1

Find the radian measure corresponding to the following degree measures:

a) 15°

b) $-37^\circ 30'$

c) 240°

d) 530°

Solution:

a) $1^\circ = \frac{\pi}{180}$ radian

$$15^\circ = \frac{\pi}{180} \times 15^\circ = \frac{\pi}{12} \text{ radian}$$

b) $1^\circ = \frac{\pi}{180}$ radian

$$-37^\circ 30' = \frac{\pi}{180} \times -37.5^\circ = \frac{-5\pi}{24} \text{ radian}$$

c) $1^\circ = \frac{\pi}{180}$ radian

$$240^\circ = \frac{\pi}{180} \times 240 = \frac{4\pi}{3} \text{ radian}$$

d) $1^\circ = \frac{\pi}{180}$ radian

$$530^\circ = \frac{\pi}{180} \times 530 = \frac{53\pi}{18} \text{ radian}$$

Question-2

Find the degree measure corresponding to the following radian measures:

a) $\frac{3}{4}$

b) -4

c) $\frac{5\pi}{3}$

d) $\frac{7\pi}{6}$

Solution:

a) $1 \text{ radian} = \frac{180^\circ}{\pi}$

$$\frac{3}{4} \text{ radian} = \frac{180^\circ}{\pi} \times \frac{3}{4} = \frac{135^\circ}{\pi} = \frac{135^\circ}{22} \times 7 = 42.95 = 42^\circ 57'$$

b) $1 \text{ radian} = \frac{180^\circ}{\pi}$

$$-4 \text{ radian} = \frac{180^\circ}{\pi} \times -4 = \frac{-720^\circ}{\pi} \times 7 = -32.72 \times 7 = -229^\circ 5' 24''$$

c) $1 \text{ radian} = \frac{180^\circ}{\pi}$

$$\frac{5\pi}{3} \text{ radian} = \frac{180^\circ}{\pi} \times \frac{5\pi}{3} = 300^\circ$$

d) $1 \text{ radian} = \frac{180^\circ}{\pi}$

$$\frac{7\pi}{6} \text{ radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210^\circ$$

Question-3

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution:

One complete revolution = 2π
 360 revolutions = $360 \times 2\pi = 720\pi$ radians / minute
 = $720\pi / 60$ radians/sec = 12π radians/sec.

Question-4

Find the degree measure of the angle subtended at the centre of a circle of diameter 200cm by an arc of length 22cm.

Solution:

Radius (r) = $(200/2)\text{cm} = 100\text{cm}$

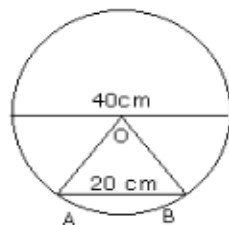
Length of an arc (l) = 22cm

$$\theta = l/r = 22/100 \text{ radians} = \frac{22}{100} \times \frac{180}{\pi} = 12^\circ 36'$$

Question-5

In a circle of diameter 40cm. The length of a chord is 20cm. Find the length of minor arc corresponding to the chord.

Solution:



Question-6

If, in two circles, arcs of the same length subtend angles of 60° and 75° at the centre, find the ratio of their radii.

Solution:

Let the radii be r_1 and r_2 . Let the angles subtend by the arcs in two circles be θ_1 and θ_2 .

$l = \theta r$ where θ is the angle, l the length of an arc and r the radius of the circle.

$$\theta_1 = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3} \text{ radian}$$

$$\theta_2 = 75^\circ = 75 \times \frac{\pi}{180} = \frac{5\pi}{12} \text{ radian}$$

$$\frac{\pi}{3} r_1 = \frac{5\pi}{12} r_2$$

$$r_1 / r_2 = \frac{\frac{5}{12}}{\frac{1}{3}} = \frac{5}{4}$$

Therefore the required ratio is 5:4.

Question-7

Find the angle in radian through which a pendulum swings if its length is 75cm and the tip described an arcs of length

- (i) 10cm
- (ii) 15cm
- (iii) 21cm

Solution:

(i) Length of the pendulum (r) = 75cm

Length of an arc (l) = 10cm

$$\theta = l/r = 10/75 \text{ radians} = 2/15 \text{ radians}$$

(ii) Length of the pendulum (r) = 75cm

Length of an arc (l) = 15cm

$$\theta = l/r = 15/75 \text{ radians} = 1/5 \text{ radians}$$

(iii) Length of the pendulum (r) = 75cm

Length of an arc (l) = 21cm

$$\theta = l/r = 21/75 \text{ radians} = 7/25 \text{ radians}$$

Question-8

Find the values of the other five trigonometric functions in the following problem: $\cos\theta = -1/2$, is quadrant III

Solution:

$\cos\theta$ and $\sec\theta$ are negative; $\tan\theta$ and $\cot\theta$ are positive.

$$\cos\theta = -\frac{1}{2}$$

$$\sin^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\therefore \sin\theta = -\frac{\sqrt{3}}{2}$$

$$\therefore \tan\theta = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$$

$$\therefore \operatorname{cosec}\theta = -\frac{1}{\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$$

$$\sec\theta = \frac{1}{-\frac{1}{2}} = -2$$

$$\cot\theta = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Question-9

Find the values of the other five trigonometric functions in the following problem: $\sin\theta = \frac{3}{5}$, θ is quadrant II

Solution:

$\sin\theta$ and $\operatorname{cosec}\theta$ positive; $\tan\theta$, $\cos\theta$, $\cot\theta$ and $\sec\theta$ all are negative.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{3}{5}\right)^2 = \frac{25-9}{25} = \frac{16}{25}$$

$$\cos\theta = -\frac{4}{5}$$

$$\therefore \tan\theta = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}$$

$$\cot\theta = \frac{-\frac{4}{5}}{\frac{3}{5}} = -\frac{4}{3}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{4}{5}} = -\frac{5}{4}$$

Question-10

Find the values of the other five trigonometric functions in the following problem: $\tan\theta = \frac{3}{4}$, θ is quadrant III

Solution:

$\sin\theta$, $\cos\theta$, $\operatorname{cosec}\theta$ and $\sec\theta$ are negative in IIIrd quadrant $\tan\theta$ and $\cot\theta$ are positive in IIIrd quadrant.

$$1 + \tan^2\theta = \sec^2\theta \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$$

$$1 + \left(\frac{3}{4}\right)^2 = \sec^2\theta$$

$$\frac{16 + 9}{16} = \frac{25}{16} = \sec^2\theta$$

$$\sec\theta = -\frac{5}{4}$$

$$\cos\theta = -\frac{4}{5}$$

$$\sin^2\theta = 1 - \left(\frac{4}{5}\right)^2 = \frac{9}{25}$$

$$\therefore \sin\theta = -\frac{3}{5}$$

$$\operatorname{cosec}\theta = -\frac{5}{3}$$

$$\cot\theta = \frac{4}{3}$$

Question-11

Find the values of the other five trigonometric functions in the following problem: $\sec\theta = 13/5$, θ lies in fourth quadrant.

Solution:

$\sin\theta$, $\tan\theta$, $\operatorname{cosec}\theta$ and $\cot\theta$ are negative in IVth quadrant $\sec\theta$ and $\cos\theta$ are positive in IVth quadrant.

$$\sec\theta = 13/5$$

$$\cos\theta = 1/\sec\theta = 5/13$$

$$\sin^2\theta = 1 - \left(\frac{5}{13}\right)^2 = \frac{144}{169}$$

$$\therefore \sin\theta = -\frac{12}{13}$$

$$\operatorname{cosec}\theta = -\frac{13}{12}$$

$$\therefore \tan\theta = \frac{-\frac{12}{13}}{\frac{5}{13}} = -\frac{12}{5}$$

$$\cot\theta = \frac{\frac{5}{13}}{-\frac{12}{13}} = -\frac{5}{12}$$

Question-12

Find the value of the following trigonometric function: $\sin 765^\circ$

Solution:

$$\sin 765^\circ = \sin \frac{17\pi}{4} = \sin \left(4\pi + \frac{\pi}{4} \right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

Question-13

Find the value of the following trigonometric function: $\operatorname{cosec}(-1410^\circ)$

Solution:

$$\operatorname{cosec}(-1410^\circ) = -\operatorname{cosec} 1410^\circ = -\operatorname{cosec} (8\pi - 30^\circ) = \operatorname{cosec} 30^\circ = 2$$

Question-14

Find the value of the following trigonometric function: $\tan \frac{13\pi}{3}$

Solution:

$$\tan \frac{13\pi}{3} = \tan \left(4\pi + \frac{\pi}{3} \right) = \tan \frac{\pi}{3} = \sqrt{3}$$

Question-15

Find the value of the following trigonometric function: $\cot\left(-\frac{15\pi}{4}\right)$

Solution:

$$\cot\left(-\frac{15\pi}{4}\right) = -\cot\left(\frac{15\pi}{4}\right) = -\cot\left(4\pi - \frac{\pi}{4}\right) = \cot \frac{\pi}{4} = 1$$

Question-16

Prove that: $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$

Solution:

$$\begin{aligned} \text{L.H.S} &= \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} \\ &= \frac{1}{4} + \frac{1}{4} - 1 \\ &= \frac{1}{2} - 1 \\ &= -\frac{1}{2} = \text{R.H.S} \end{aligned}$$



Question-17

Prove that: $2\sin^2\frac{\pi}{6} + \operatorname{cosec}\frac{7\pi}{6} \cos^2\frac{\pi}{3} = 0$

Solution:

$$\begin{aligned}\text{L.H.S} &= 2\sin^2\frac{\pi}{6} + \operatorname{cosec}\frac{7\pi}{6} \cos^2\frac{\pi}{3} \\ &= 2 \times \frac{1}{4} + \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) \times \frac{1}{4} \\ &= \frac{1}{2} - \operatorname{cosec}\frac{\pi}{6} \times \frac{1}{4} \\ &= \frac{1}{2} - 2 \times \frac{1}{4} \\ &= 0 = \text{R.H.S}\end{aligned}$$

Question-18

Prove that: $3\cos^2\frac{\pi}{4} + \sec\frac{2\pi}{3} + 5\tan^2\frac{\pi}{3} = \frac{29}{2}$

Solution:

$$\begin{aligned}\text{L.H.S} &= 3\cos^2\frac{\pi}{4} + \sec\left(\pi - \frac{\pi}{3}\right) + 5\tan^2\frac{\pi}{3} \\ &= 3 \times \frac{1}{2} - \sec\frac{\pi}{3} + 5\tan^2\frac{\pi}{3} \\ &= \frac{3}{2} - 2 + 5 \times (\sqrt{3})^2 \\ &= \frac{3}{2} - 2 + 15 \\ &= 13 + \frac{3}{2} \\ &= \frac{29}{2} \\ &= \text{R.H.S}\end{aligned}$$

Question-19

Prove that: $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

Solution:

$$\begin{aligned}\text{L.H.S} &= \cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} \\ &= \cot^2\frac{\pi}{6} + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\tan^2\frac{\pi}{6} \\ &= \cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{\pi}{6} + 3\tan^2\frac{\pi}{6} \\ &= (\sqrt{3})^2 + 2 + 3\left(\frac{1}{\sqrt{3}}\right)^2 \\ &= 3 + 2 + 1 \\ &= 6 \\ &= \text{R.H.S}\end{aligned}$$

Question-20

Prove that: $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

Solution:

$$\begin{aligned}\text{L.H.S} &= 2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} \\ &= 2\sin^2\left(\pi - \frac{\pi}{4}\right) + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} \\ &= 2\sin^2\frac{\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} \\ &= 2 \times \frac{1}{2} + 2 \times \frac{1}{2} + 2 \times 4 \\ &= 1 + 1 + 8 \\ &= 10 \\ &= \text{R.H.S}\end{aligned}$$

Question-21

Show that:

$$\cos 70^\circ \cos 10^\circ + \sin 70^\circ \sin 10^\circ = \frac{1}{2}$$

Solution:

We know that

$$\begin{aligned}\cos(\theta - \phi) &= \cos\theta \cos\phi + \sin\theta \sin\phi \therefore \text{L.H.S} = \cos 70^\circ \cos 10^\circ + \sin 70^\circ \sin 10^\circ \\ &= \cos(70^\circ - 10^\circ) \\ &= \cos 60^\circ \\ &= \frac{1}{2}\end{aligned}$$

Question-22

Show that:

$$\cos 130^\circ \cos 40^\circ + \sin 130^\circ \sin 40^\circ = 0$$

Solution:

We know that

$$\begin{aligned}\cos(\theta - \phi) &= \cos\theta \cos\phi + \sin\theta \sin\phi \therefore \text{L.H.S} = \cos 130^\circ \cos 40^\circ + \sin 130^\circ \sin 40^\circ \\ &= \cos(130^\circ - 40^\circ) \\ &= \cos 90^\circ \\ &= 0\end{aligned}$$



Question-23

Show that:

$$\sin(40^\circ + \theta)\cos(10^\circ + \theta) - \cos(40^\circ + \theta)\sin(10^\circ + \theta) = \frac{1}{2}$$

Solution:

We know that

$$\begin{aligned}\sin(\theta - \phi) &= \sin\theta \cos\phi - \cos\theta \sin\phi \text{ L.H.S} = \sin(40^\circ + \theta)\cos(10^\circ + \theta) - \cos(40^\circ + \theta)\sin(10^\circ + \theta) \\ &= \sin 30^\circ \\ &= \frac{1}{2} \\ &= \text{R.H.S}\end{aligned}$$

Question-24

Prove that:

$$\cos\left(\frac{\pi}{4} - \theta\right)\cos\left(\frac{\pi}{4} - \phi\right) - \sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{\pi}{4} - \phi\right) = \sin(\theta + \phi)$$

Solution:

We know that

$$\begin{aligned}\cos(\theta + \phi) &= \cos\theta \cos\phi - \sin\theta \sin\phi \text{ L.H.S} = \cos\left(\frac{\pi}{4} - \theta\right)\cos\left(\frac{\pi}{4} - \phi\right) - \sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{\pi}{4} - \phi\right) \\ &= \cos\left(\frac{\pi}{4} - \theta + \frac{\pi}{4} - \phi\right) \\ &= \cos\left(\frac{\pi}{2} - (\theta + \phi)\right) \\ &= \sin(\theta + \phi) \\ &= \text{R.H.S}\end{aligned}$$

Question-25

Prove that:

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

Solution:

$$\begin{aligned}\text{L.H.S} &= \frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} \\ &= \frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4} \tan x} \\ &= \frac{1 + \tan x}{1 - \tan x} \\ &= \left(\frac{1 + \tan x}{1 - \tan x}\right)^2 \\ &= \text{R.H.S}\end{aligned}$$

Question-26

Prove that :

$$\frac{\cos(\pi + \theta)\cos(-\theta)}{\sin(\pi - \theta)\cos\left(\frac{\pi}{2} + \theta\right)} = \cot^2 \theta$$

Solution:

$$\begin{aligned}\text{L.H.S} &= \frac{\cos(\pi + \theta)\cos(-\theta)}{\sin(\pi - \theta)\cos\left(\frac{\pi}{2} + \theta\right)} \\ &= \frac{(-\cos\theta)\cos\theta}{\sin\theta(-\sin\theta)} \\ &= \cot^2\theta = \text{R.H.S}\end{aligned}$$

Question-27

$$\cos\theta + \sin(270^\circ + \theta) - \sin(270^\circ - \theta) + \cos(180^\circ + \theta) = 0.$$

Solution:

$$\begin{aligned}\text{L.H.S} &= \cos\theta + \sin(270^\circ + \theta) - \sin(270^\circ - \theta) + \cos(180^\circ + \theta) \\ &= \cos\theta - \cos\theta + \cos\theta - \cos\theta = 0 = \text{R.H.S}\end{aligned}$$

Question-28

$$\cos\left(\frac{3\pi}{2} + \theta\right)\cos(2\pi + \theta) \left[\cot\left(\frac{3\pi}{2} - \theta\right) + \cot(2\pi + \theta)\right] = 1.$$

Solution:

$$\begin{aligned}\text{L.H.S} &= \cos\left(\frac{3\pi}{2} + \theta\right)\cos(2\pi + \theta) \left[\cot\left(\frac{3\pi}{2} - \theta\right) + \cot(2\pi + \theta)\right] \\ &= (-\sin\theta)(-\cos\theta)(\tan\theta + \cot\theta) \\ &= \sin\theta \cos\theta (\tan\theta + \cot\theta) \\ &= \sin^2\theta + \cos^2\theta = 1 = \text{R.H.S}\end{aligned}$$

Question-29

$$\sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x.$$

Solution:

$$\begin{aligned}\text{L.H.S} &= \sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x \\ &= \cos[(n+1)x - (n+2)x] \\ &= \cos(-x) \\ &= \cos x \\ &= \text{R.H.S}\end{aligned}$$



Question-30

Find the value of:

(i) $\cos 210^\circ$

(ii) $\sin 225^\circ$

(iii) $\tan 330^\circ$

(iv) $\cot (-315^\circ)$

Solution:

$$(i) \cos 210^\circ = \cos (180^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$(ii) \sin 225^\circ = \sin (270^\circ + 45^\circ) = \cos 45^\circ = -\frac{1}{\sqrt{2}}$$

$$(iii) \tan 330^\circ = \tan (360^\circ - 30^\circ) = -\tan 30^\circ = -\frac{1}{\sqrt{3}}$$

$$(iv) \cot (-315^\circ) = \cot (315^\circ) = \cot (360^\circ - 45^\circ) = \cot 45^\circ = 1$$

Question-31

Find the value of $\tan(\alpha + \beta)$, given that

$$\cot \alpha = \frac{1}{2}, \alpha \in \left(\pi, \frac{3\pi}{2}\right) \text{ and } \sec \beta = -\frac{5}{3}, \beta \in \left(\frac{\pi}{2}, \pi\right)$$

Solution:

$$\cot \alpha = \frac{1}{2}, \alpha \in \left(\pi, \frac{3\pi}{2}\right)$$

$$\text{and } \sec \beta = -\frac{5}{3}, \beta \in \left(\frac{\pi}{2}, \pi\right)$$

$$\tan \beta = -\frac{4}{3}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$= \frac{2 - \frac{4}{3}}{1 - 2 \times \frac{-4}{3}}$$

$$= \frac{\frac{2}{3}}{\frac{11}{3}}$$

$$= \frac{2}{11}$$

Question-32

Prove the following identity:

$$\sin(150^\circ + x) + \sin(150^\circ - x) = \cos x$$

Solution:

We know that

$$2\sin\theta \cos\phi = \sin(\theta + \phi) + \sin(\theta - \phi)$$

$$\text{L.H.S} = \sin(150^\circ + x) + \sin(150^\circ - x)$$

$$= 2\sin 150^\circ \cos x$$

$$= 2\sin(180^\circ - 30^\circ) \cos x$$

$$= 2\sin 30^\circ \cos x$$

$$= 2(1/2)\cos x$$

$$= \cos x$$

$$= \text{R.H.S}$$

Question-33

Prove the following identity:

$$\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$$

Solution:

We know that

$$-2\sin\theta \sin\phi = \cos(\theta + \phi) - \cos(\theta - \phi)$$

$$\text{L.H.S} = -2\sin \frac{3\pi}{4} \sin x = -2 \sin\left(\pi - \frac{\pi}{4}\right) \cos x = -2 \sin \frac{\pi}{4} \sin x = -2 \times \frac{1}{\sqrt{2}} \sin x = -\sqrt{2} \sin x =$$

$$\text{R.H.S}$$

Question-34

Prove the following identity:

$$\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = \sqrt{2} \cos x$$

Solution:

We know that

$$2\cos\theta \cos\phi = \cos(\theta + \phi) + \cos(\theta - \phi)$$

$$\text{L.H.S} = 2\cos \frac{\pi}{4} \cos x = 2 \times \frac{1}{\sqrt{2}} \cos x = \sqrt{2} \cos x = \text{R.H.S}$$

Question-35

$$\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$$

Solution:

$$\begin{aligned}\text{L.H.S} &= \sin 2x + 2\sin 4x + \sin 6x \\&= 2\sin \frac{6x+2x}{2} \cos \frac{6x-2x}{2} + 2\sin 4x \\&= 2\sin 4x \cos 2x + 2\sin 4x \\&= 2\sin 4x(\cos 2x + 1) \\&= 4\sin 4x \cos^2 x \\&= \text{R.H.S}\end{aligned}$$

Question-36

$$\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$$

Solution:

$$\begin{aligned}\text{L.H.S} &= \sin^2 6x - \sin^2 4x \\&[\text{Using the formulae } 2\sin^2 x = 1 - \cos 2x] \\&= \frac{1 - \cos 12x}{2} - \frac{1 - \cos 8x}{2} \\&= -\frac{1}{2}(\cos 12x - \cos 8x) \\&= -\frac{1}{2}[\cos(10x + 2x) - \cos(10x - 2x)] \\&[\text{Using the formulae } -2\sin\theta \sin\phi = \cos(\theta + \phi) - \cos(\theta - \phi)] \\&= \frac{1}{2} \times 2 \sin 10x \sin 2x \\&= \sin 10x \sin 2x\end{aligned}$$

Question-37

$$\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$$

Solution:

$$\begin{aligned}\text{L.H.S} &= \cos^2 2x - \cos^2 6x \\&[\text{Using the formulae } 2\cos^2 x = 1 + \cos 2x] \\&= \frac{1 + \cos 4x}{2} - \frac{1 + \cos 12x}{2} \\&= -\frac{1}{2}(\cos 12x - \cos 4x) \\&= -\frac{1}{2}[\cos(8x + 4x) - \cos(8x - 4x)] \\&[\text{Using the formulae } -2\sin\theta \sin\phi = \cos(\theta + \phi) - \cos(\theta - \phi)] \\&= \frac{1}{2} \times 2 \sin 8x \sin 4x \\&= \sin 4x \sin 8x\end{aligned}$$

Question-38

$$\cos 7x + \cos 5x + \cos 3x + \cos x = 4 \cos x \cos 2x \cos 4x$$

Solution:

$$\text{L.H.S} = \cos 7x + \cos x + \cos 5x + \cos 3x$$

$$= \cos(4x + 3x) + \cos(4x - 3x) + \cos(4x + x) + \cos(4x - x)$$

[Using the formula $2\cos\theta \cos\phi = \cos(\theta + \phi) + \cos(\theta - \phi)$]

$$= 2\cos 4x \cos 3x + 2\cos 4x \cos x$$

$$= 2\cos 4x (\cos 3x + \cos x)$$

$$= 2\cos 4x (\cos 3x + \cos x)$$

[Using the formula $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$]

$$= 4\cos 4x \cos \frac{3x+x}{2} \cos \frac{3x-x}{2}$$

$$= 4\cos 4x \cos 2x \cos x$$

$$= \text{R.H.S}$$

Question-39

$$\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$$

Solution:

$$\text{L.H.S} = \cot 4x (\sin 5x + \sin 3x)$$

[Using the formula $\sin 5x + \sin 3x = 2 \sin \frac{5x+3x}{2} \cos \frac{5x-3x}{2}$]

$$= 2\cot 4x \sin \frac{5x+3x}{2} \cos \frac{5x-3x}{2}$$

$$= 2\cot 4x \sin \frac{8x}{2} \cos \frac{2x}{2}$$

$$= 2\cot 4x \sin 4x \cos x$$

$$= 2\cos 4x \cos x$$

$$\text{R.H.S} = \cot x (\sin 5x - \sin 3x)$$

[Using the formula $\sin 5x - \sin 3x = 2 \cos \frac{5x+3x}{2} \sin \frac{5x-3x}{2}$]

$$= 2\cot x \cos \frac{5x+3x}{2} \sin \frac{5x-3x}{2}$$

$$= 2\cot x \cos \frac{8x}{2} \sin \frac{2x}{2}$$

$$= 2\cos 4x \cos x$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

Question-40

$$\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x.$$

Solution:

$$\tan 3x = \tan(2x + x)$$

$$= (\tan 2x + \tan x) / (1 - \tan 2x \tan x)$$

$$\tan 3x(1 - \tan 2x \tan x) = \tan 2x + \tan x$$

$$\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$$

$$\therefore \tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x.$$

Question-41

Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = \frac{-\sin 2x}{\cos 10x}$

Solution:

$$\begin{aligned} \text{L.H.S} &= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} \\ &= \frac{-2 \sin \frac{9x+5x}{2} \sin \frac{9x-5x}{2}}{2 \cos \frac{17x+3x}{2} \sin \frac{17x-3x}{2}} \\ &= \frac{-\sin \frac{14x}{2} \sin \frac{4x}{2}}{\cos \frac{20x}{2} \sin \frac{14x}{2}} \\ &= \frac{-\sin 7x \sin 2x}{\cos 10x \sin 7x} \\ &= \frac{-\sin 2x}{\cos 10x} \\ &= \text{R.H.S} \end{aligned}$$

Question-42

Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Solution:

$$\begin{aligned} \text{L.H.S} &= \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} \\ &= \frac{2 \sin \frac{5x+3x}{2} \cos \frac{5x-3x}{2}}{2 \cos \frac{5x+3x}{2} \cos \frac{5x-3x}{2}} \\ &= \frac{\sin \frac{8x}{2}}{\cos \frac{8x}{2}} \\ &= \frac{\sin 4x}{\cos 4x} \\ &= \tan 4x \\ &= \text{R.H.S} \end{aligned}$$

Question-43

Prove that $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$

Solution:

$$\begin{aligned}\text{L.H.S} &= \frac{\sin x - \sin y}{\cos x + \cos y} \\&= \frac{2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}}{2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}} \\&= \frac{\sin \frac{x-y}{2}}{\cos \frac{x-y}{2}} \\&= \tan \frac{x-y}{2} \\&= \text{R.H.S}\end{aligned}$$

Question-44

Prove that $(\sin A + \sin 3A)/(\cos A + \cos 3A) = \tan 2A$

Solution:

$$\begin{aligned}\text{L.H.S} &= (\sin A + \sin 3A)/(\cos A + \cos 3A) = 2\sin 2A \cos A / 2\cos 2A \cos A = \tan 2A \\&= \text{R.H.S}\end{aligned}$$

Question-45

Prove that $\frac{\sin x + \sin y}{\cos x + \cos y} = \tan \frac{x+y}{2}$

Solution:

$$\begin{aligned}\text{L.H.S} &= \frac{\sin x + \sin y}{\cos x + \cos y} \\&= \frac{2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}}{2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}} \\&= \frac{\sin \frac{x+y}{2}}{\cos \frac{x+y}{2}} \\&= \tan \frac{x+y}{2} \\&= \text{R.H.S}\end{aligned}$$



Question-46

Prove that $(\tan 5\theta + \tan 3\theta)/(\tan 5\theta - \tan 3\theta) = 4\cos 2\theta \cos 4\theta$

Solution:

$$\begin{aligned}\text{L.H.S} &= \frac{\frac{\sin 5\theta \cos 3\theta + \sin 3\theta \cos 5\theta}{\cos 5\theta \cos 3\theta}}{\frac{\sin 5\theta \cos 3\theta - \sin 3\theta \cos 5\theta}{\cos 5\theta \cos 3\theta}} \\&= \frac{\sin 8\theta}{\frac{\cos 5\theta \cos 3\theta}{\sin 2\theta}} \\&= \frac{\sin 8\theta \sin 2\theta}{\cos 5\theta \cos 3\theta}\end{aligned}$$

$[\sin(A+B) = \sin A \cos B + \cos A \sin B \text{ and } \sin(A-B) = \sin A \cos B - \cos A \sin B]$

$$\begin{aligned}&= \frac{\sin 8\theta \sin 2\theta}{\cos 5\theta \cos 3\theta} \\&= \frac{2 \sin 4\theta \cos 4\theta \sin 2\theta \cos 2\theta}{\cos 5\theta \cos 3\theta} \quad (\sin 2\theta = 2 \sin \theta \cos \theta) \\&= \frac{2 \times 2 \sin 2\theta \cos 2\theta \sin 2\theta \cos 2\theta}{\cos 5\theta \cos 3\theta} \quad (\sin 2\theta = 2 \sin \theta \cos \theta) \\&= 4 \cos 2\theta \cos 4\theta \\&= \text{R.H.S}\end{aligned}$$

Hence proved

Question-47

Prove that $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$

Solution:

$$\begin{aligned}\text{L.H.S} &= \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} \\&= \frac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x} \\&= \frac{2 \cos \frac{3x+x}{2} \sin \frac{3x-x}{2}}{\cos 2x}\end{aligned}$$

[Using the identity $\cos 2x = \cos^2 x - \sin^2 x$ and $\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$]

$$\begin{aligned}&= \frac{2 \cos 2x \sin x}{\cos 2x} \\&= 2 \sin x \\&= \text{R.H.S}\end{aligned}$$

Question-48

Prove that $(3\sin A - \sin 3A)^{2/3} + (3\cos A + \cos 3A)^{2/3} = 4^{2/3}$

Solution:

$$\begin{aligned}\text{L.H.S} &= (3\sin A - \sin 3A)^{2/3} + (3\cos A + \cos 3A)^{2/3} \\&= [3\sin A - (3\sin A - 4\sin^3 A)]^{2/3} + [3\cos A + (4\cos^3 A - 3\cos A)]^{2/3} \\&= (4\sin^3 A)^{2/3} + [4\cos^3 A]^{2/3} \\&= 4^{2/3}(\sin^2 A + \cos^2 A)^{2/3} \\&= 4^{2/3} \cdot 1^{2/3} \\&= 4^{2/3} \\&= \text{R.H.S}\end{aligned}$$

Question-49

Prove that $\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

Solution:

$$\begin{aligned}\text{L.H.S} &= \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} \\&= \frac{2\sin \frac{7x+5x}{2} \cos \frac{7x-5x}{2} + 2\sin \frac{9x+3x}{2} \cos \frac{9x-3x}{2}}{2\cos \frac{7x+5x}{2} \cos \frac{7x-5x}{2} + 2\cos \frac{9x+3x}{2} \cos \frac{9x-3x}{2}} \\&= \frac{2\sin 6x \cos x + 2\sin 6x \cos 3x}{2\cos 6x \cos x + 2\cos 6x \cos 3x} \\&= \frac{2\sin 6x(\cos x + \cos 3x)}{2\cos 6x(\cos x + \cos 3x)} \\&= \tan 6x \\&= \text{R.H.S}\end{aligned}$$

Question-50

Prove that $\cos 4x = 1 - 8\sin^2 x \cos^2 x$.

Solution:

$$\text{L.H.S} = \cos 4x = 1 - 2\sin^2 2x = 1 - 2(2\sin x \cos x)^2 = 1 - 8\sin^2 x \cos^2 x = \text{R.H.S}$$

Hence proved.